

Random manifolds and quantum gravity

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The non-perturbative, lattice field theory approach towards the quantization of Euclidean gravity is reviewed. Included is a tentative summary of the most significant results and a presentation of the current state of art.

1. Preamble

The last year was relatively quiet in this field. There are new results, interesting for experts, less for the audience of a plenary session. Therefore, I shall rather try to give you a tentative idea of what has been achieved since my plenary talk of 1988 and where we do stand now. For lack of space I quote only the recent papers, omitting those quoted in the monograph [1].

2. Is Euclidean gravity worth attention?

The true subject of this talk is the statistical mechanics of random manifolds, a subject of intrinsic interest. The partition function has the structure

$$Z = \sum_{\text{geometry}} \sum_{\text{matter}} e^{-\text{action}} \quad (1)$$

The novelty is the summation over geometry, which is considered to be dynamical. When the action depends only on the intrinsic geometry, the geometrical degrees of freedom are the topology and the metric of the manifolds. The summation over topologies being beyond control, one sums over all inequivalent metrics at fixed topology.

Euclidean quantum gravity belongs to statistical mechanics. Its relevance for the gravity theory at $d > 2$ is controversial. But it is, at least, an interesting theoretical laboratory, with its general covariance, perturbative non-renormalizability and bottomless action.

3. The appeal of dynamical triangulations

The most promising discretization of Euclidean gravity rests on the idea of dynamical triangulations : the sum over geometries is defined to be the sum over all possible ways of gluing together equilateral simplices.

It is worth emphasizing the a priori beauty of this approach:

- The arbitrariness of vertex labeling replaces the reparametrization invariance in the continuum.
- The concept of a background metric never appears.
- No recourse is made to perturbation theory. Non-perturbative renormalization can be carried out using geometrical observables to set the scale.
- On a lattice the action is not bottomless. But for $d > 2$ the most probable manifolds do *not* correspond to the bottom of the action.

4. The glory in two dimensions

In 2d the discrete model of pure gravity can be solved exactly. The results match those obtained in the continuum². Actually, the analytic power of the discrete theory often exceeds that of the continuum formalism. The discussion can in many cases be extended to 2d gravity coupled to conformal matter fields. Let me briefly sample some highlights:

- In 2d one has

$$\#triangulations(A) \sim A^{\gamma-3} e^{\kappa_c A}, \quad (2)$$

²The connection between the dynamical triangulations and the Liouville theory is further studied in [2], where the authors attempt to reconstruct the Liouville field in the discrete framework.

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where A is the total area and γ is the string susceptibility exponent. Summing over triangulations with two marked points separated by the geodesic distance r defines an invariant "two-point" function $G(r)$, whose explicit calculation for pure gravity is a great success of the theory. The scaling properties of $G(r)$ determine the two basic critical exponents: the Hausdorff dimension d_H of space-time, which turns out to be $d_H = 4$ (not the naive 2!) and $\gamma = -1/2$.

- An exact solution has been found [3] for the discrete R^2 gravity in 2d (here R denotes the scalar curvature). It has been proved that the infrared behavior of the system is that of the standard Liouville gravity for any finite R^2 coupling³. The R^2 term flattens the surfaces locally, but at large scales they always look alike.

- The equivalence between dynamical triangulations and matrix models can be used to derive a number of results [5]. E.g. the string susceptibility exponent can be calculated for all unitary models where 2d gravity is coupled to conformal matter fields⁴: writing $c = 1 - 6/m(m+1)$ one finds for spherical topology $\gamma = -1/m$. Of course, this holds at the critical point⁵. The topological expansion enables one to extend this result to higher genera⁶.

- The "gravitational dressing" of the scaling exponents is under control. It depends on c only⁷. The celebrated example is that of the Ising model on a randomly triangulated manifold: the critical exponents have been calculated exactly and differ from the classical Onsager ones!

- It was long unclear whether there is any in-

³However, it is well known that the surfaces become crumpled, when this coupling is large negative. The transition Liouville \rightarrow crumpled seems to be a cross-over.

⁴The result was first found for some discrete models, then generalized in the continuum framework and eventually rederived using matrix models.

⁵The geometrical scaling at $c \neq 0$ is not yet understood. Numerical simulations yield $d_H \approx 4$ for $0 \leq c \leq 1$. See also [4].

⁶Attempts to go beyond the perturbative approach, via the so-called double scaling limit and the associated differential equations [5], did not provide, as yet, the expected insight into the non-perturbative physics of fluctuating topologies.

⁷A discussion of scaling exponents observed on quenched geometries thermalized with a "wrong" value of c can be found in [6].

intermediate phase between Liouville gravity and branched polymers, above the $c = 1$ barrier. The answer seems to be found by in [7], using a renormalization group argument: the critical behavior at $c > 1$ is generically that of branched polymers, but finite size effects are exponentially enhanced when $c \rightarrow 1$. Furthermore, David's argument suggested the existence of phenomena, which have eventually been found numerically [8].

The aim of these examples is to illustrate the claim that 2d "gravity" is at present the best, if not the only example of fully fledged quantum geometry. Other approaches to quantum gravity, including the most advertized ones, not only do not tell us much about the microscopic geometry of space-time, they do not offer yet a suitable framework to ask many relevant questions.

5. The boom of baby universes

One of the most interesting results in the field is the discovery that it is very unlikely for a generic random manifold to make small fluctuations around some more or less smooth average configuration. If one lets it fluctuate freely, there are bubbles, called baby universes, growing out of it. Further, there are baby universes growing on baby universes and so on. The final structure is a fractal.

Using combinatorial arguments and (2) one can estimate the average number of baby universes with a given volume B in a manifold of volume A to be :

$$n(B, A) \sim A[(1 - \frac{B}{A})B]^{\gamma-2}, \quad B < A/2, \quad (3)$$

so that γ also controls the fractality of the manifolds⁸. From (3) one finds easily that the number of baby universes carrying a *finite* fraction of the total volume behaves like A^γ . Clearly, dramatic things happen when $\gamma > 0$: large baby universes proliferate and the manifolds degenerate into polymer-like structures.

Recently, Ambjørn, Loll and collaborators [9] have devised an alternative, model of 2d quantum gravity, where the creation of baby universes

⁸This result also holds for $d > 2$ provided (2) is true, with A being the volume.

does not occur. The consequence is that most of the results listed in sect. 4 are gone: there is no anomalous scaling, $d_H = 2$, Ising model critical exponents take Onsager's values etc.

Introducing a chemical potential for minimal neck baby universes and using it to enhance the weight of configurations with large number of babies one produces an ensemble whose string susceptibility exponent is that of branched polymers [8].

One could multiply such examples. They suggest that the dynamics of the Euclidean quantum gravity is to large extent controlled by the dynamics of the baby universes. Anticipating on the discussion to follow let me remark that one of basic problems at $d > 2$ is that we do not know how to tame the process of baby universe creation.

6. The improving tools

Reporting about the progress made in this field one should mention the impressive development of the tools employed. In short, people interested in random geometries have now at their disposal the complete toolbox of a perfect lattice theorist: local and ergodic algorithms, a nonlocal algorithm [10], renormalization group techniques, especially the node decimation one [11], long strong coupling series, also for $d > 2$ [12].

7. $d > 2$: What are telling the computers ?

7.1. Discrete Einstein-Hilbert action

Our understanding of $d > 2$ rests on numerical simulations. For $d = 3$ it was rapidly established that the pure gravity model has two phases, separated by a first order transition. In one phase the manifolds are elongated, resembling branched polymers, in the other they are crumpled. A similar transition was found in 4d, but data were compatible with it being a continuous one. Precise simulations at large volumes have shown that this transition is, in fact, of first order too [13]. This was confirmed by other studies [14]. This year 4d simulations were carried out with the so-called degenerate triangulations. In [15] the double peak in the "energy" histogram is already seen at $N_4 = 4000$, while in [13] it is only observed at

$N_4 = 32000$. This opens the possibility of studying the effect in a wide volume interval.

Incidentally, the data of [15] also strengthen the numerical evidence that the number of triangulations is in 4d exponentially bounded. This bound, for $d > 2$, is our community's contribution to topology!

The dynamics of the transition is by now elucidated [16]-[17]. At finite volume the transition occurs in two steps : first some singular vertices are formed, then these vertices condense and in the crumpled phase one finds a sub-singular link connecting two highly singular vertices ⁹.

7.2. Models with modified action

Attempts were made to soften the transition by modifying the action. The simplest such modification follows the old proposal by Brugmann and Marinari [18]: one weights the triangulations with

$$weight\ factor = \prod_{j=1}^{N_0} o(v_j)^\alpha, \quad (4)$$

where $o(v_j)$ is the order of the vertex v_j . Alternatively one can put the triangle order instead, this does not make much difference. Another modification, which at the end of the day turned out to give very similar results, is discussed in sect. 10.

The phase diagram was already discussed at length by Thorleifsson last year. At sufficiently negative values of α a new phase, baptized "crinkled" in [19], appears. It looks smoother than the crumpled one, since the Hausdorff dimension and γ are finite and $\gamma < 0$. However, it seems that with increasing volume the transition crumpled \rightarrow crinkled runs towards large values of the Einstein term coupling, where $\langle N_0 \rangle / N_4 \rightarrow 1/4$. This corresponds to the kinematic boundary. The crinkled phase is also infested with sub-singular vertices.

The dynamical triangulations near the kinematic boundary were studied this year in 3d, using both the strong coupling series and the Monte Carlo simulations [20]. The transition branched

⁹The structure of the crumpled phase described here is not universal. With degenerate triangulations one rather observes in the crumpled phase a gas of sub-singular vertices [15].

→ crinkled looks continuous, perhaps of third order. In the crinkled phase $\gamma < 0$, but $d_H = \infty$ or ≈ 2 , depending on whether it is measured on the triangulation or its dual, a feature hardly compatible with the existence of a sensible thermodynamic limit.

8. $d > 2$: The same story by backgammon players

The structure of the phase diagram can be qualitatively understood within an exactly solvable model inspired by the backgammon game [21]. The idea is to replace the sum over triangulations by the sum over weighted partitions of vertex orders, the weight being $\propto o(v)^\alpha$. Set $r = N_0/N_4$. In the thermodynamical limit the model has generically two phases : for $\kappa < \kappa_c$ one has $r = 0$ and a singular vertex with order $\sim N_4$, while for $\kappa > \kappa_c$ the value of r is finite and all vertex orders are bounded. This mocks the transition crumpled → branched.

For large enough negative α the system jumps at $\kappa = \kappa_c$ from $r = 0$ to $r = 1/4$, and the order of the most singular vertex drops suddenly (but remains $\sim N_4$). This is similar to the transition crumpled → crinkled¹⁰.

9. A generic instability?

No phase identified at $d > 2$ is a serious candidate for a physical space-time. The alternative, crumpled or branched, also occurs in 2d at $c > 1$ ¹¹. I am tempted to formulate the following conjecture: *the generic random manifolds are unstable*. If true, this would be a very interesting result in statistical mechanics. Somewhat frustrating viewed from the quantum gravity perspective,

¹⁰ Strictly speaking, for $-2 < \alpha < -1$ the transition crumpled → branched becomes continuous. This prediction has not really been checked. But it is unlikely to be verified. The predictions of the model should not be taken too literally. E.g. simulations with degenerate triangulations in 3d indicate that in the crumpled phase $\langle N_0 \rangle / N_3$ approaches a *finite value* ≈ 0.02 when $N_3 \rightarrow \infty$ [22]. Also, in 2d, the model does not see the $c = 1$ barrier

¹¹ The Liouville phase appears to be an exception. This can perhaps be understood: the system gravity+matter with $c < 1$ seems over-constrained (for a recent discussion see [2]).

although it cannot be excluded that the instabilities are a lattice artifact and that the physical space-time is recovered as one approaches some as yet undiscovered fixed point. My feeling is that we are rather missing some important part of the puzzle. It is a big challenge to discover it. Or, if this were the case, to prove that the constructive, lattice approach is inadequate.

10. The miseries of a brilliant idea

One would like to control the dynamics of baby universes. In 2d and for $c > 1$, there occurs a condensation of metric singularities ("spikes") which can be avoided by moving c below unity. In [23] it was suggested that a similar phenomenon might occur in 4d. But, in 4d the sign of matter contributions to the conformal factor effective action is opposite to that found in 2d and one can hope stabilizing the manifolds by *adding* conformal matter fields. One should mention, however, that in 4d the idea is less founded than in 2d.

A numerical experiment testing that is described in [24], reporting encouraging results. But it was found later, in [19], that the model with extra matter fields and that with measure modified à la Brugmann-Marinari are essentially equivalent. Hence, the absence of polymerization seen in [24] has here an origin very different from the expected one.

Thus the idea has failed. But the way it failed is interesting. The introduction of gauge matter fields produces a *local* modification of the action. The simplest explanation of this fact is that the correlations between field fluctuations become short ranged. If so, the gauge fields have no chance to do the job they were supposed to do.

11. Wishful thoughts about the future

Let me just mention briefly a few ideas which start receiving attention.

In standard dynamical triangulations one sums over *all* simplicial complexes. Perhaps one should restrict the support of the Feynman integral. Can one figure out what a relevant constraint could be? This question is the starting point of [9]. They start in the Lorentzian regime in

2d and consider triangulations endowed with a causal structure. They eventually go to the Euclidean formulation, but with a dramatically reduced class of admissible triangulations. Baby universes are prohibited and the manifolds are much smoother than in Liouville gravity. Actually, they look too smooth, one would like quantum gravity to me more entertaining, but perhaps this will be the case for $d > 2$.

Another almost unexplored avenue is SUSY. It is essential in string theory. Perhaps it is a necessary ingredient of any sensible quantum gravity theory? Perhaps with SUSY one can avoid the localization of matter fields and revive the idea reviewed in sect. 10¹². Another hint: according to [25] the $c = 1$ barrier disappears in $N = 2$ world-sheet SUSY models. One immediate problem: what SUSY, target space¹³ or world sheet¹⁴? The latter is necessarily broken on the lattice. Perhaps too strongly broken to have significant consequences? But would the target space SUSY be enough? Implementing SUSY is a notoriously difficult problem of fundamental importance. Models in 2d should help developing intuition on the back-reaction of SUSY on geometry.

In conclusion, much has been achieved, but further surprises are not expected if one does not go beyond what became the common lore. We must be "bold, yet bolder, even most bold". This is what Danton said in 1792. One year later ...

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¹²There exists a class of one-dimensional solvable models of disorder, where SYSY and delocalization are related. Could this be a hint? These models are admittedly very special.

¹³An attempt in this direction is being made in [26]

¹⁴A small step in this direction is made in [27], where fermions are put on a random manifold.

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